



# Federated In-Context Prompt Selection for Multi-Modal 3D Dental Imaging: A Theoretical Framework with Privacy-Preserving Guarantees

## Abstract

Vision-language models show remarkable capabilities in medical imaging analysis, yet their deployment in federated healthcare environments faces key challenges in privacy preservation, data heterogeneity, and adversarial robustness. We present FedDental3D-ICL, a theoretical framework for federated in-context prompt learning that enables privacy-preserving collaboration across healthcare institutions without sharing sensitive patient data or model parameters. Our framework introduces four core algorithmic contributions: Multi-Modal Prompt Space (MMPS) abstraction unifying visual and textual prompt representations across 2D and 3D medical imaging modalities; Cross-Modal Prompt Alignment (CMPA) ensuring semantic consistency through information-theoretic contrastive objectives; Hierarchical Multi-Modal Optimization (HMMO) providing theoretical convergence guarantees for non-convex federated objectives; and Byzantine-Resilient Cross-Modal Aggregation (BRCMA) with differential privacy bounds. Our theoretical analysis suggests potential convergence rates of  $O(1/\sqrt{T})$ , theoretical communication complexity bounds of  $O(K \log |P|)$  compared to traditional  $O(K \cdot d)$ , and  $(\epsilon, \delta)$ -differential privacy guarantees with optimal composition bounds. While this work establishes comprehensive mathematical foundations, empirical validation and practical implementation remain important directions for future research.

## Introduction

Federated, privacy-preserving multimodal AI is urgently needed in healthcare, particularly in areas like dental imaging where large, diverse, and sensitive patient datasets are indispensable for advancing diagnosis and treatment. Yet the risks of traditional data sharing are profound: between 2009 and 2025 there have been 6,759 U.S. healthcare data breaches of 500 or more records, exposing or impermissibly disclosing the protected health data of 846,962,011 individuals. In 2023 alone, 725 such large breaches were reported, exposing over 133 million records. These breaches set new records in both number of incidents and number of records breached. At the same time, medical imaging analysis stands at a critical juncture where the transformative potential of vision-language models (VLMs) collides with the immutable constraints of healthcare data governance. Recent advances in VLMs have shown unprecedented multimodal reasoning capabilities, yet their deployment in real-world healthcare environments exposes a fundamental paradox: the models that show the greatest promise require precisely the type of large-scale, cross-institutional data sharing that regulatory frameworks explicitly prohibit. This tension is more than a technical bottleneck—it is a systemic barrier that prevents the medical community from fully leveraging state-of-the-art AI while upholding the privacy guarantees essential to patient trust and compliance.

## Research Questions

**RQ1.** How can federated multi-modal prompt learning frameworks be designed to reconcile the privacy-utility paradox, enabling vision-language models to achieve clinically useful diagnostic performance in dental imaging without direct data sharing?

**RQ2.** To what extent can hierarchical optimization strategies in federated settings address the statistical heterogeneity of dental imaging data across institutions, ensuring stable convergence and generalizable model performance?

**RQ3.** Can privacy-preserving, Byzantine-resilient aggregation mechanisms safeguard against both adversarial clients and regulatory non-compliance while maintaining efficiency and robustness in large-scale multimodal healthcare training?

## Framework

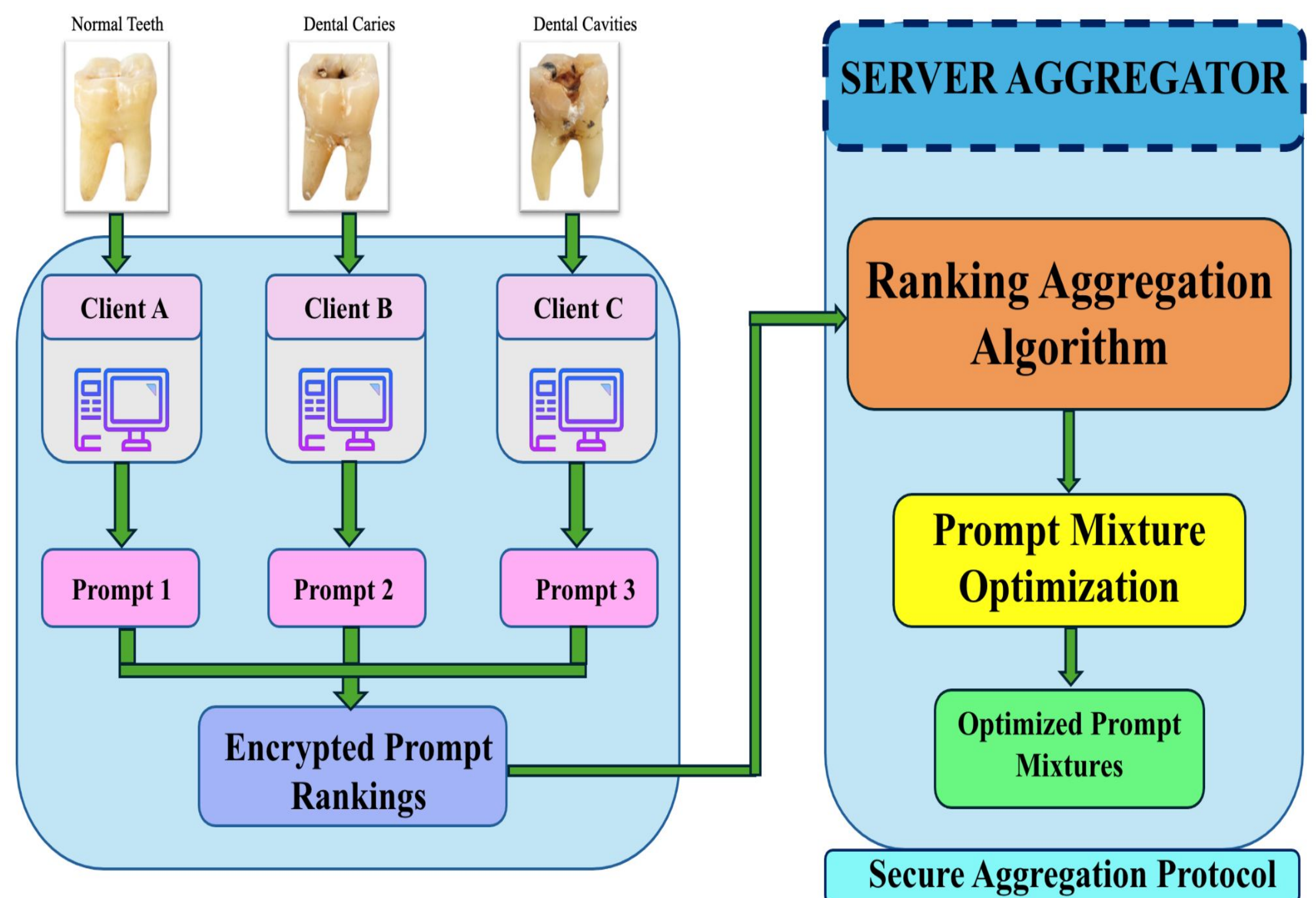


Fig. 1. FedDental3D-ICL System Architecture showing multi-modal data flow across federated dental institutions with privacy-preserving prompt exchange.

## Proposed Algorithm

### Algorithm 1 Multi-Modal Prompt Space Construction (MMPS)

**Require:** Raw prompts  $P_v, P_t, P_{3D}$ ; contrastive parameters  $(\tau, \text{batch\_size})$

**Ensure:** Unified embeddings  $\psi(p_v, p_t, p_{3D})$ , learned fusion function  $F$

```
1: Initialize embedding functions  $\phi_v, \phi_t, \phi_{3D}$  with random weights
2: Initialize fusion network  $F$  with Xavier initialization
3: ▷ Phase 1: Individual modality embedding learning
4: for each modality  $m \in \{v, t, 3D\}$  do
5:   for epoch = 1 to  $E_1$  do
6:     Sample batch of prompts  $\{p_m^{(i)}\}$ 
7:     Compute embeddings  $z_m^{(i)} = \phi_m(p_m^{(i)})$ 
8:     Update  $\phi_m$  via contrastive loss minimization
9:   end for
10: end for
11: ▷ Phase 2: Cross-modal fusion learning
12: for epoch = 1 to  $E_2$  do
13:   Sample multi-modal triplets  $(p_v^{(i)}, p_t^{(i)}, p_{3D}^{(i)})$ 
14:   Compute modality embeddings  $z_v^{(i)} = \phi_v(p_v^{(i)})$ ,  $z_t^{(i)} = \phi_t(p_t^{(i)})$ ,  $z_{3D}^{(i)} = \phi_{3D}(p_{3D}^{(i)})$ 
15:   Compute fused embedding  $\psi^{(i)} = F(z_v^{(i)}, z_t^{(i)}, z_{3D}^{(i)})$ 
16:   Update  $F$  via contrastive loss on  $\psi^{(i)}$ 
17: end for
18: return  $\psi(p_v, p_t, p_{3D}), F$ 
```

### Algorithm 2 Hierarchical Multi-Modal Optimization (HMMO)

**Require:** Global prompt candidates  $P_{\text{global}}$ , participation probability  $p_m$

**Ensure:** Optimal prompt distribution  $\theta$ , client adaptations  $\{\theta_k\}$

```
1: Initialize global prompt parameters  $\theta^{(0)}$ 
2: for round  $t = 1$  to  $T$  do
3:   Select subset of clients  $S_t \subseteq [K]$  with probability  $p_m$ 
4:   for each client  $k \in S_t$  do
5:     Evaluate prompt candidates on local data:  $q_k(p) = \text{quality}(M(x_k, p), y_k)$  for  $p \in P_{\text{global}}$ 
6:     Generate local prompt ranking:  $r_k = \text{argsort}(q_k, \text{descending} = \text{True})$ 
7:     Compute alignment statistics:  $a_k = \text{cross\_modal\_alignment}(r_k)$ 
8:     Send encrypted  $(r_k, a_k)$  to server
9:   end for
10:   Aggregate rankings via Byzantine-resilient mechanism:  $\theta^{(t)} = \text{BRCMA}(\{r_k, a_k\}_{k \in S_t})$ 
11: end for
12: return  $\theta^{(T)}, \{\theta_k^{(T)}\}$ 
```

### Algorithm 3 Byzantine-Resilient Cross-Modal Aggregation with Privacy-Preserving Selection (BRCMA-PMMS)

**Require:** Client rankings  $\{r_k\}$ , privacy parameters  $(\epsilon, \delta)$ , prompt set  $P$

**Ensure:** Aggregated prompt parameters  $\theta$ , privacy-preserving selection

```
1: Initialize global prompt parameters  $\theta^{(0)}$ 
2: for round  $t = 1$  to  $T$  do
3:   Collect client rankings  $\{r_k^{(t)}\}$  and quality scores  $\{q_k^{(t)}\}$ 
4:   Apply exponential mechanism to select top prompts:
5:      $p_{\text{select}}(p) \propto \exp\left(\frac{\epsilon \cdot q(D, p)}{2\Delta q}\right)$ 
6:   Filter out Byzantine clients using cross-modal consistency check:
7:      $S_t^{\text{valid}} = \{k : \text{consistency}(r_k, \{r_j\}_{j \neq k}) > \tau_{\text{byz}}\}$ 
8:   Aggregate valid client rankings using median-based robust estimator
9:   Add calibrated Gaussian noise for  $(\epsilon, \delta)$ -differential privacy
10:   Update global prompt distribution:  $\theta^{(t)} = \text{weighted\_aggregate}(S_t^{\text{valid}})$ 
11: end for
12: return  $\theta^{(T)}$ 
```